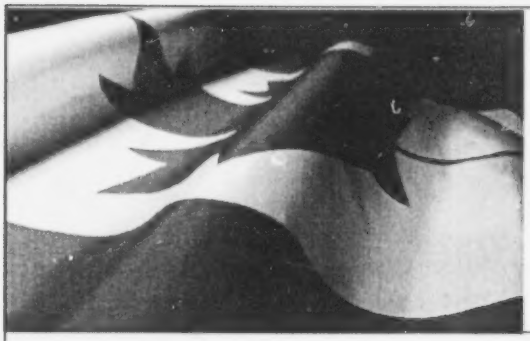


Department of Finance
Ministère des Finances

Working Paper
Document de travail

Borrowing Short- or Long-Term: Does the Government Really Face a Trade-off?

by

Patrick Georges*

Working Paper 2003-16

* I wish to thank without implicating, Doug Hostland for inspiring this research and valuable comments. I also thank Benoît Robidoux, Pierre Gilbert, Rob Stewart, Jean-François Fillion, Jenna Robbins, Chris Matier, and Jeff Danforth for useful comments and discussions on earlier drafts. Please address all comments to Georges.Patrick@fin.gc.ca.

Working Papers *are circulated in the language of preparation only, to make analytical work undertaken by the staff of the Department of Finance available to a wider readership. The paper reflects the views of the authors and no responsibility for them should be attributed to the Department of Finance. Comments on the working papers are invited and may be sent to the author(s).*

Les **Documents de travail** *sont distribués uniquement dans la langue dans laquelle ils ont été rédigés, afin de rendre le travail d'analyse entrepris par le personnel du Ministère des Finances accessible à un lectorat plus vaste. Les opinions qui sont exprimées sont celles des auteurs et n'engagent pas le Ministère des Finances. Nous vous invitons à commenter les documents de travail et à faire parvenir vos commentaires aux auteurs.*

Abstract

This paper considers a broader fiscal policy framework in public debt management decisions by examining the interrelations between interest rates and the economy. This approach has the potential to result in a new optimal debt maturity structure. A shorter debt structure is generally less expensive and *may* also contribute to stabilize the budget balance as interest rates tend to decline when the economy is weak.

Résumé

Ce papier envisage le cadre plus large de la politique budgétaire dans les décisions de gestion de la dette publique tout en examinant les interrelations entre les taux d'intérêts et l'économie. Cette approche peut potentiellement résulter en une nouvelle structure optimale de maturité de la dette. Une structure plus à court terme est généralement moins coûteuse et *peut* contribuer à stabiliser la balance budgétaire si les taux d'intérêt tendent à baisser quand l'économie est fragile.

104-03321/

Introduction

Different perspectives exist on the choice of the maturity structure of public debt. Debt managers have traditionally analysed this choice as a trade-off between *low* and *stable* debt service cost. The policy implication is that a risk-averse government should issue a "prudent" (*i.e.*, long-term) debt maturity structure to limit exposure to ongoing refinancing requirements and unexpected interest rate changes.

However, debt management can be analysed from another perspective as a component of a more general problem involving fiscal planning under uncertainty. Under this approach the preferred debt maturity structure is determined by its influence on the overall budget balance, rather than on debt service cost alone.

We show in this paper that a shorter debt maturity structure is *less* expensive on average and may be *less* risky from the point of view of the overall budget balance if demand shocks prevail over the business cycle. The policy implication is that a fiscal planner is likely to favour a debt maturity structure shorter than the one chosen by the debt manager if he/she believes that demand shocks are likely to prevail relative to other shocks (*e.g.*, interest rate, inflation, and supply shocks).

This paper should not be viewed as an attempt to contrast "debt managers" with "fiscal planners" but to emphasize that a broader macroeconomic view of debt management may shed a new light that could serve the field of debt management.

We assume throughout this study that the government of Canada does not engage in opportunistic borrowing through a financing strategy that attempts to pick ideal market conditions. The government of Canada does not attempt to save by shortening debt maturity when the yield curve is particularly steep and by lengthening maturity when the yield curve is flat or inverted. It is assumed in this paper that once the government selects a debt maturity structure, it maintains it for the remaining fiscal planning horizon (ten years). This assumption seems to be supported by a variety of public documents (*e.g.*, Department of Finance-Canada, 2001) which stress that the general debt management strategy is not based on a particular interest rate outlook.

The plan of this paper is as follows. Section 1 shows how debt managers have traditionally analysed the decision about the maturity structure of the debt. Sections 2 and 3 show how a fiscal planning perspective, by embracing a more global perspective, may considerably change our way of thinking about this issue. Section 4 lists some caveats to our analysis and Section 5 concludes.

1. A debt manager's perspective

Opponents of short-term borrowing often argue that a shorter maturity structure would increase the risks of debt management. While short-term borrowing may be cheaper on average given that the yield curve is typically upward sloping, it would be more risky because short rates are more volatile than long rates. On the other hand, borrowing long would be more costly on average but less risky.

Hence, debt managers traditionally claim that the choice of the maturity structure of the debt involves a trade-off. Borrowing for shorter terms decreases the expected debt service cost, but at the cost of an increase in the volatility of the debt service distribution. This view implies that the first and second moments of the debt service distribution (*i.e.*, the mean and the variance) must be balanced against each other when choosing a debt maturity structure.

To shed some light on the nature of this trade-off we have used a stochastic macro model of the Canadian economy developed in the Department of Finance [see Hostland (2001) and Georges and Moreau (2002) for a description of key elements of this model]. Whereas purely financial models used by debt managers focus exclusively on financial issues in modelling government debt (*i.e.*, interest rates and the government debt portfolio), treating the primary budget balance as exogenous, our more global approach addresses these financial issues and also models the primary balance endogenously. This implies that macroeconomic variables such as real GDP and inflation, as well as the link to financial variables such as the term structure of interest rates must be explicitly modelled. Our model includes a monetary policy rule that sets, on a quarterly basis, short term interest rates to offset deviations of expected inflation rate over the coming year from its target. Finally, the fiscal authority plans a budget surplus of $4 billion every year, which reflects a prudence factor. The financial authority reformulates its budget plans (expected tax revenues and spending) once a year taking into account economic shocks. Our stochastic model generates probability distributions for each variable, including the probability distributions of the debt service cost over a specific time horizon (10 years).

We have simulated our model of the Canadian economy under several debt maturity structures and report the results obtained under four of them. See Appendix 1 for an illustration of how the debt maturity structure is computed. The first debt financing strategy roughly approximates the current maturity structure of the federal debt. According to this strategy, 64% of the public debt is fixed, that is *roughly* one-third of the public debt has to be refinanced every year. In the jargon of debt managers, the fixed-floating ratio is said to be 64%. Two alternatives examine "shorter" debt maturity structures which correspond to fixed-floating ratios of 60% and 50%. A fourth financing strategy examines the potential consequences of increasing the fixed-floating ratio to 70%.

For these four debt maturity structures Figure 1 illustrates the average or expected debt service cost, and Figure 2 illustrates their associated risk. (Appendix 1 gives some detail

on the computation of the average debt service cost.) These two figures taken together illustrate the trade-off between average cost and risk for the Canadian economy.

Figure 1 illustrates that the average debt service cost over a ten-year horizon will be lower the shorter the debt maturity structure (the lower the fixed-floating ratio). For example, moving from a 64% to a 50% fixed-floating ratio entails moving from an average debt service cost given by the dark solid line (roughly the average expected cost under the current maturity structure) to the average cost given by the alternatively short-and-long dashed line. This policy change would therefore lower the debt service cost by roughly $1 billion on a yearly basis.[1]

However, by borrowing more at short-term maturities, the debt manager faces more frequent refinancing requirements at new short-term interest rates. Because short rates are typically more volatile than long rates, this implies a riskier distribution for the debt service cost. Figure 2 illustrates this result with 90% confidence intervals for the debt service cost for alternative debt maturity structures. The probabilistic ceiling associated with each debt maturity structure is a measure of the "cost at risk", a "worst-case" scenario. Only 5% of all simulated debt service costs were actually worst than this "ceiling". Clearly, moving to a 50% fixed-floating ratio shifts up the probabilistic ceiling or cost at risk. In other words, a shorter debt maturity structure increases the risk of having to cope with particularly high ex post debt service costs.

Although the cost at risk measure given in Figure 2 shows that a lower fixed-floating ratio is more risky, we can gain a better appreciation of the risk involved by thinking of the "worst-case" deviation from the expected (average) debt service cost (under normal market conditions). Figure 3 illustrates this measure referred as relative cost at risk. To interpret the results, consider for example "year 5" of the planning horizon. Under a 64% fixed-floating ratio there is only a small probability of 5% of seeing a deviation from the average debt service cost by $5.5 billion or more. Shifting to a 50% fixed-floating ratio would increase this deviation from the mean debt service cost to $ 7.1 billion (or more). This represents a non-negligible increase in risk.

Figures 1 and 2 (or 3) therefore summarize for the Canadian economy, the trade-off that is examined by debt managers. Shorter debt maturity structures typically imply lower average debt cost but a larger variance and thus a riskier distribution. If we remain

---

[1] One may ask why "year 1" in Figure 1 seems atypical in the sense that it does not generate lower debt service cost for smaller fixed-floating ratios. In our model, the last historical year is 2000 and thus "year 1", the first simulated year, is 2001. It is the case, however, that the average yield curve is flatter than typical in 2001 because the positive historical aggregate demand shock of 2000 has led to an inflation rate higher than targeted, which forces the central bank in maintaining atypical high short interest rates in 2001. The implication is that a maturity structure that is biased in favor of shorter-term instruments (and thus, which implies a large share of public debt refinancing in 2001) cannot deliver the typical lower average debt service cost because the refinancing of the debt is done at these atypical high short rates. This is a nice illustration of the risk of facing unexpected high rates incurred with a shorter debt maturity structure as will be illustrated shortly with the discussion surrounding Figure 2.

agnostic about the government's preferences, then it is difficult to conclude further.[2] However, a risk-averse government (or debt manager) would in fact prefer a "prudent" structure, that is a long-term debt maturity structure to limit exposure to unexpected interest rate changes and ongoing refinancing requirements.

Suppose for example that debt managers accept, for the next fiscal year of the planning horizon, a small risk (a probability of 5%) that the debt service cost unexpectedly deviates by $3 billion or more from its planned level. The relative cost at risk measure is thus set equal to $3 billion for the next fiscal year. Figure 3 shows that a fixed-floating ratio of 50% is unacceptable for debt managers given their tolerance for risk. Indeed, this debt maturity structure suggests a relative cost at risk in excess of $3 billion for the first year. In other words, the probability of observing an unexpected deviation of $3 billion or more from the average debt service cost is higher than 5% under this financing strategy.

Figure 3 also shows that given the tolerance for risk expressed by debt managers, fixed-floating ratios of 60%, 64%, and 70% would all be acceptable. Under these conditions it makes sense to choose the 60% fixed-floating financing strategy because it is less costly on average over the long term than the alternatives (64% or 70% ratios).[3]

---

[2] As stated by Bolder (2002): "we do not have a clear understanding of the government's preferences for uncertainty in terms of both variance of expected cost and time diversification. The result is that instead of finding an optimal solution we must seek to understand the nature of the trade-offs between expected cost, variance and time diversification that can potentially be made."

[3] See the Debt Management Strategy 2003-2004, Department of Finance Canada, for a more detailed description of that approach.

Figure 1. Average Debt Service Cost Under Alternative Debt Maturity Structures

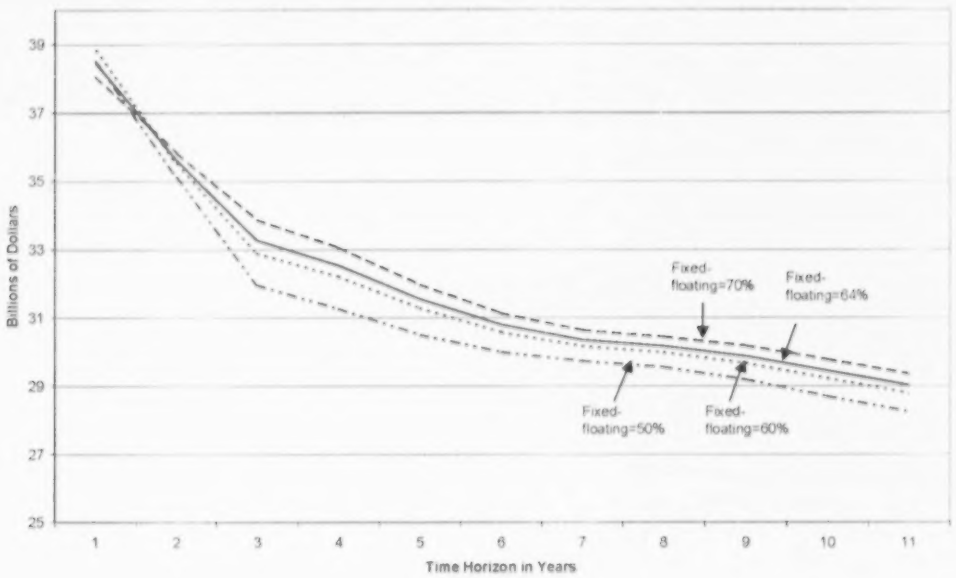


Figure 2. 90% Confidence Intervals for Debt Service Cost

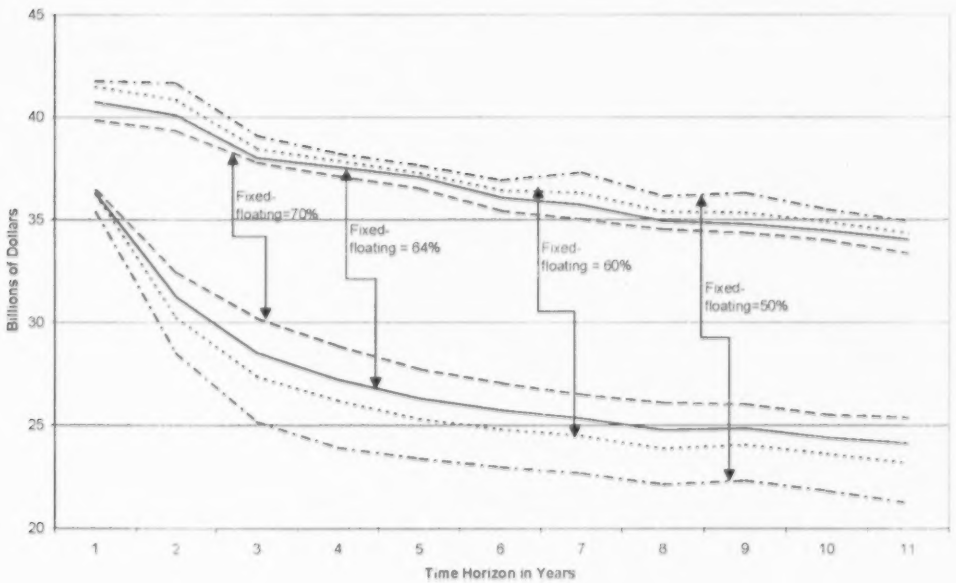
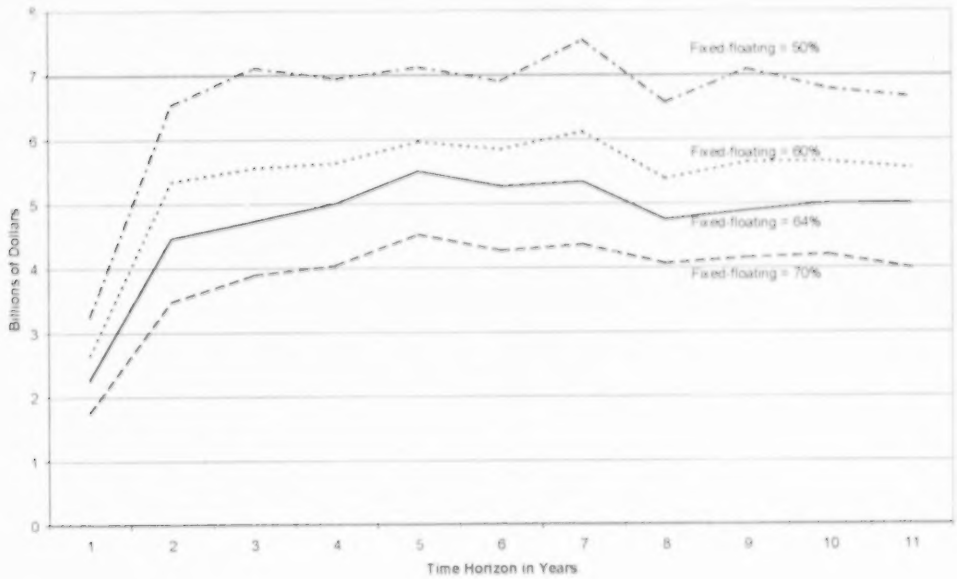


Figure 3. Relative Cost at Risk



2. A fiscal planner's perspective

The argument

We argue in this section that the Federal government in the recent past may have implicitly revealed its preference across debt maturity structures, by its strong emphasis on (or aversion to) the risk of a deficit outcome. This aversion, however, may result in a *more* risky debt service distribution, and thus a shorter debt maturity structure! Although this seems to be a paradox, the objective of this section is to show why we generate this result in our model.

The emphasis placed by the fiscal authority on the probability of a deficit outcome reveals (among other things) that a fiscal planner is more concerned with the variability of the budget balance as a whole than with the variability of one of its component, the debt service burden.

In order to see why this is the case, it is helpful to give a symbolic definition of the budget balance as:

$$(1) \qquad \begin{aligned} bal &= tax - ps - ds \\ &= pbal - ds \end{aligned}$$

where *bal* is the budget balance, *tax* is the tax revenue, *ps* is the program spending, *ds* is the debt service, and $pbal = tax - ps$ is the primary balance. The variability of the overall balance is thus given by:

$$(2) \qquad var(bal) = var(pbal) + var(ds) - 2\, cov(pbal, ds)$$

where *var* (.) and *cov* (. , .) are respectively the variance and covariance.

The variability of debt service adds directly to the variability of the budget balance. For example, Bolder (2002) says that: "a financing strategy consisting of entirely short-term debt will increase the variability of government expenditures [*presumably ds in equation* (2)] and hence the variability of the government's financial requirements [*presumably bal in equation* (2)]."[4] However, this view must assume that either the covariance between the primary balance and the debt service is negative or zero; if positive it must be sufficiently small to be negligible, or if large at times, sufficiently unstable through time that it might be more appropriate to abstract from this covariance effect when developing a strategy for the maturity structure of the debt.[5]

With all of these caveats in mind, economic analysis suggests that economic and financial developments may have a simultaneous effect on debt service costs and fiscal expenditures and revenues. For example, in the case of a contractionary aggregate

---

[4] Square brackets added to the original quotation.

[5] Note that the caveat on the instability of the covariance is different from the caveat that we address at the end of the paper, which suggests that a debt financing strategy can be self-defeating.

demand shock, the monetary authority responds by lowering interest rates to push back expected inflation to its target. While the monetary response lowers debt service costs, the contraction in aggregate demand lowers tax revenues and thus moves the primary budget balance toward a deficit position. These simultaneous effects act to stabilize the overall balance. In terms of equation (2), the monetary policy response induces a positive covariance between changes in debt service costs and the primary budget balance, which reduces the variance of the overall balance and thus reduces the risk of a deficit outcome.[6]

What does that have to do with the choice of the maturity structure of the public debt? The main argument is that the impact of the central bank's response on the volatility of the budget balance depends on the debt maturity structure. If this structure is biased towards long-term bonds, changes in short rates implemented by the central bank in response to the economic context have little impact on the overall debt service (because only a small share of the debt is to be refinanced in this particular period) and thus little stabilizing impact on the budget balance. However, if the maturity structure is biased in favour of shorter-term bonds and Treasury Bills, short rates set by the central bank will strongly affect debt service costs (because a large share of the public debt has to be refinanced at these new rates), which stabilizes the budget balance.

We show in Figures 4a and 4b how the elements of equation (2) change for each year of the fiscal planning horizon when the fixed-floating ratio is reduced from 64% (Figure 4a) to 50% (Figure 4b) in our model. These figures illustrate that our model generates a positive covariance [such that $-2cov(pbal,ds)$ is negative] for the simulated fiscal planning horizon and that the positive covariance term in equation (2) increases by more than the increase in the variance of the debt service as the fiscal authority reduces the fixed-floating ratio.

In conclusion, the stabilizing effect of the covariance term on the overall budget balance is more effective the shorter the debt maturity structure is, and this results in a lower probability of a deficit outcome. In our model, this lower probability is due *exclusively* to a more stable variance of the overall budget balance. The distribution of the overall balance stays centered on the same mean ($4 billion) under all debt maturity structures, and thus the debt reduction plan of the fiscal authority is unaffected by this choice (public debt is reduced by $4 billion on average every year under all scenarios).[7]

---

[6] Recall that the probability of a deficit outcome is the probability that the overall balance falls in deficit. Thus *ceteris paribus*, a smaller variance for the balance implies a smaller probability of a deficit outcome.

[7] In other words, the probability distribution of the overall budget balance does not shift as the debt maturity structure is reduced even if the average debt service cost is lower. This necessarily implies that the fiscal authority increases program spending or reduces taxes accordingly. If, after choosing a shorter maturity structure, the fiscal authority did not increase program spending (or reduce taxes) to offset the average saving in debt service costs, the distribution of the overall budget balance would not only have a smaller variance, it would also be centered on a higher mean, which would furthermore reduce the probability of a deficit outcome. Some results of Hermanutz and Poitras (2001) are based on this effect.

Figure 4a. Fixed-floating ratio: 64%

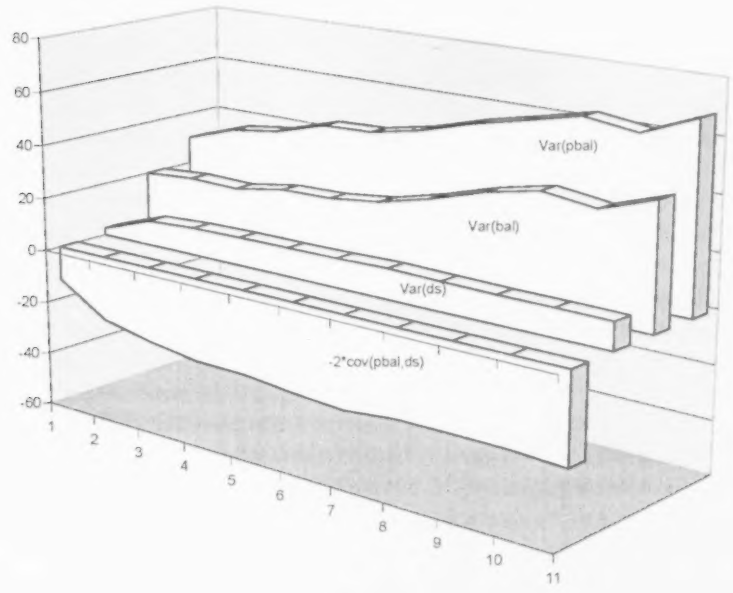
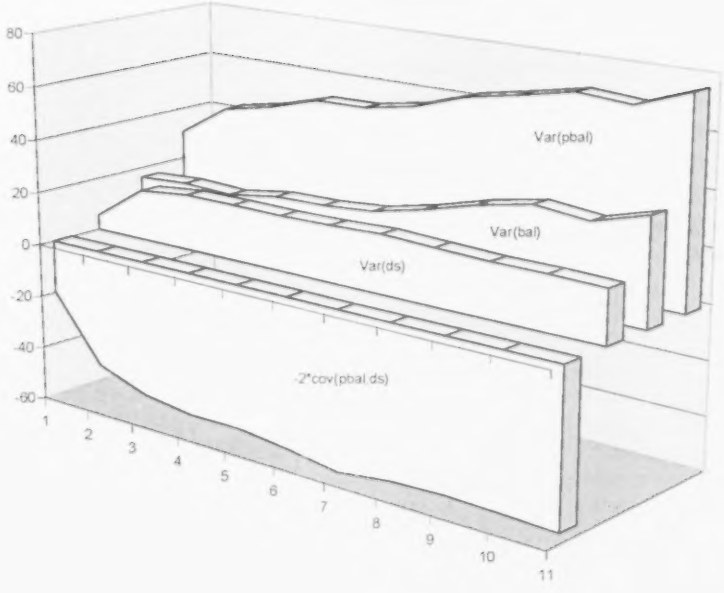


Figure 4b. Fixed-floating ratio: 50%

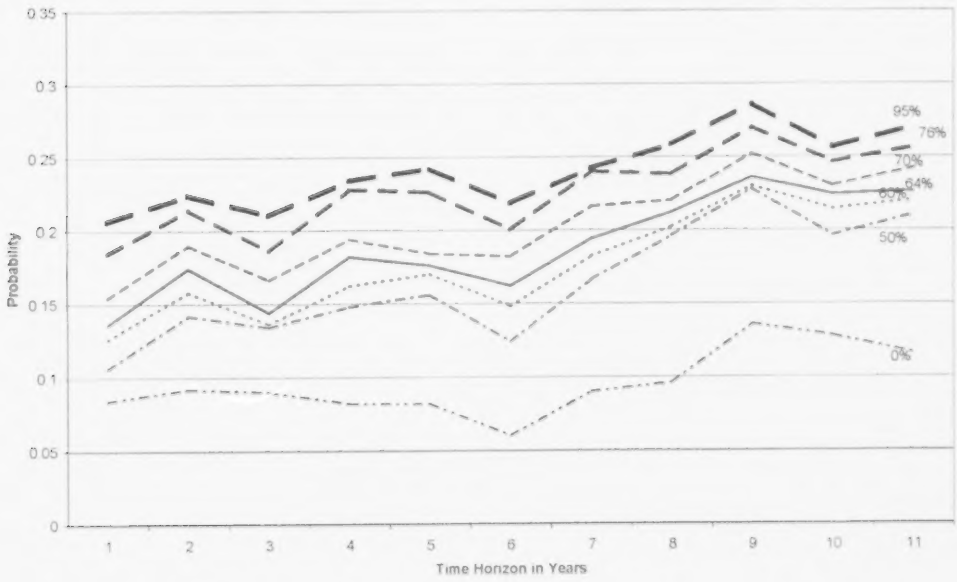


Does the stabilizing effect of the covariance have a significant impact on the probability of a deficit outcome in our simulations? Figure 5 illustrates for different debt maturity structures, the probability of a deficit outcome over a 10-year interval. Moving from a 64% to a 50% debt maturity structure would decrease the probability of a deficit outcome by approximately 3 percentage points.

This is a relatively small decrease in the probability of a deficit outcome, but the main point of this analysis lies elsewhere. Indeed, we are now ready to explain the paradox given earlier that a prudent government (fiscal planner), in our model, would prefer a riskier debt service distribution and thus a shorter debt maturity structure.

First, a lower fixed-floating ratio (that is a shorter debt maturity structure) is less risky from the point of view of the *overall* budget balance (as just shown in Figure 5) even if it is more risky from the debt service distribution alone (see Figures 2 or 3). Second, such a strategy is also *less* expensive on average (lower expected debt service) as shown in Figure 1.

Figure 5. Probability of a Deficit Outcome Under Alternative Debt Maturity Structures



In conclusion, although a trade-off exists from the perspective of a debt manager, a fiscal planning approach to debt management, by embracing a more global perspective, considerably changes our way of thinking at this issue. In the context of our model and given its calibration, a shorter debt maturity structure is less expensive on average and also *less* risky from the point of view of the overall budget balance. It turns out that a

prudent fiscal authority, in our model, should always select the shortest among the set of feasible debt maturity structures (that is, the set of structures that are acceptable from an operational point of view, given for example the constraint of liquidity of government bond issues). In Section 4 we will raise the issue of whether moving to a shorter maturity structure is an exploitable or self-defeating strategy.

The nature of the shocks

An important issue that we did not explicitly mention when developing the intuition behind the "covariance effect" is the nature of the shocks hitting the economy. The relevance of the source of the shocks is highlighted in a recent OECD Report on Interest-Rate Risk Management of Government Debt (2002).

To illustrate our previous results we have referred to aggregate demand shocks. These shocks directly affect the output gap and, through a Phillips curve, they affect the inflation rate, which triggers a central bank's reaction that pushes the inflation back to target with the help of its policy interest rate. These shocks are always associated with a positive covariance between the economic activity and the interest rate and thus between the primary balance and the debt service.

However, there are other sources of shocks which could lead to a negative covariance. For example, an inflation/price shock (unrelated to aggregate demand) will trigger a monetary response to impending inflationary pressures. The increase in interest rates following a "positive" inflation shock will itself lower the output gap or aggregate demand, moving the primary balance towards a deficit position (lower tax revenues and higher cyclical program spending). Hence, inflation shocks in our model generate a negative covariance between economic activity and interest rates and thus between the primary balance and debt service cost. Exogenous interest rate shocks (unrelated to aggregate demand), due for example to an increase in interest rates in the U.S., would increase debt service costs and lower the output gap and thus tax revenues, leading to a negative covariance between the primary balance and debt service.[8] Finally, a negative supply shock that pushes output below potential could generate a recession *and* inflationary pressures. The central bank would increase interest rates to avoid the inflationary pressures at a time when tax revenues (and thus the primary balance) are low, generating a negative covariance between the primary balance and debt service.

Our model generates aggregate demand shocks as well as inflation and interest rate shocks leading to the (positive or negative) covariances described above. Although our model generates "supply" shocks, they do not lead to the negative covariance just described. A negative shock to potential output decreases potential output but leads to an immediate and proportional decrease in economic activity without generating an output gap (demand above potential) and thus without generating inflationary pressure. Because

[8] By "exogenous" interest rates shocks, it is meant increases (or decreases) in interest rates that are unrelated to an output gap position. Shocks to international (U.S.) rates may put upward pressures on Canadian rates. Within Canada, a lack of credibility of monetary policy may require additional upward pressures on domestic rates.

shocks to potential output are assumed to be permanent, demand has to adjust to this new lower potential output and this adjustment is assumed to be immediate in our model. Hence, there is no reason for the central bank to react by revising its policy interest rate. The decrease in potential output, and thus economic activity, decreases the primary balance through lower tax revenues, but with no change in interest rates. Hence, in our model, this kind of "supply" shock implies a zero covariance, not a negative one.

In conclusion, in our model, aggregate demand shocks will lead to a positive covariance, inflation and interest rate shocks will lead to a negative covariance and shocks to potential output lead to a zero covariance. In terms of equation (2), a positive covariance implies that a government should borrow for shorter terms if the covariance effect is sufficiently large to offset the increased variance of the debt service. A negative covariance, on the other hand, implies that a government that wants to minimize the risk of a deficit outcome should borrow for longer terms because, in absolute value, the covariance term will be lower the larger the fixed-floating ratio is, while the variance of debt service will also decrease.

The importance of the covariance between the debt service cost and the primary balance (or tax revenues) when deciding for a fixed-floating ratio can also be illustrated as follows. A fiscal planner will be concerned with the risk that debt service cost rises at a time when the budget is already weak due to other factors, for example, due to lower tax revenues than expected because of a recession. If a negative covariance is highly likely as shown in Figure 6a, a longer maturity structure for the debt will protect the government by avoiding some of the upward pressures on debt service costs when the economy is weak and thus when tax revenues are low.[9] The higher average debt service cost associated with a longer debt maturity structure is the premium that the government would be willing to pay to obtain this protection in bad times.

On the other hand, if a positive covariance between the primary balance and debt service is more likely to occur (Figure 6b), a long term maturity structure would negatively impact the government because, when the economy is weak, it could not benefit from the additional downward pressures on debt service costs that a shorter maturity structure would offer. In this case, the longer debt maturity structure would imply that the government would pay a premium to *avoid* the possibility of lower debt service costs when the economy is weak! In this context, a shorter maturity structure is a win-win strategy because the government would pay a lower debt service on average and be protected in bad times by additional downward pressures on debt service cost.

---

[9] A positive (negative) covariance between debt service and tax revenue is schematically represented by a straight line with a positive (negative) slope. The slope is always larger (in absolute value) for the shorter debt maturity structure to reflect the fact that debt service costs in this case are more sensitive to changes in interest rates.

Figure 6a: Risk of an upward pressure on debt service cost when the economy is weak

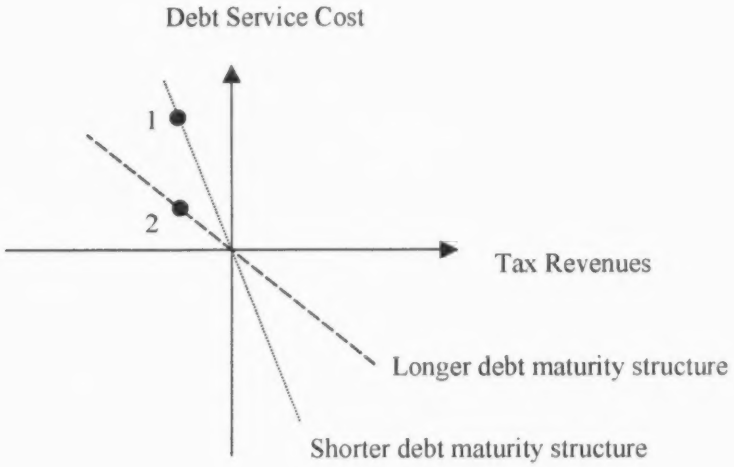
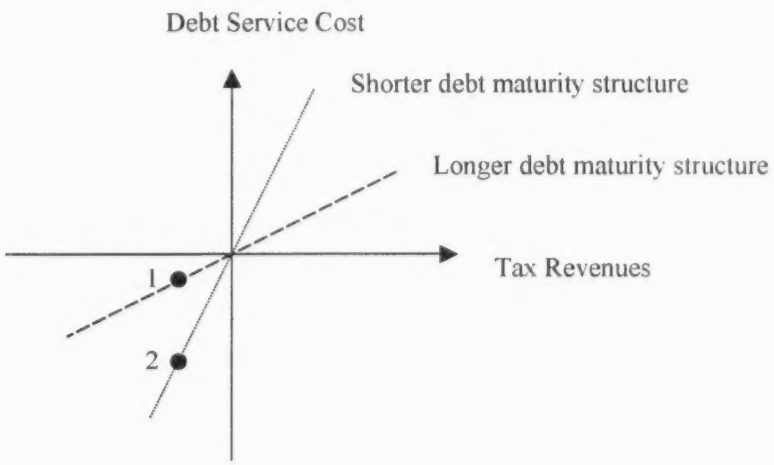


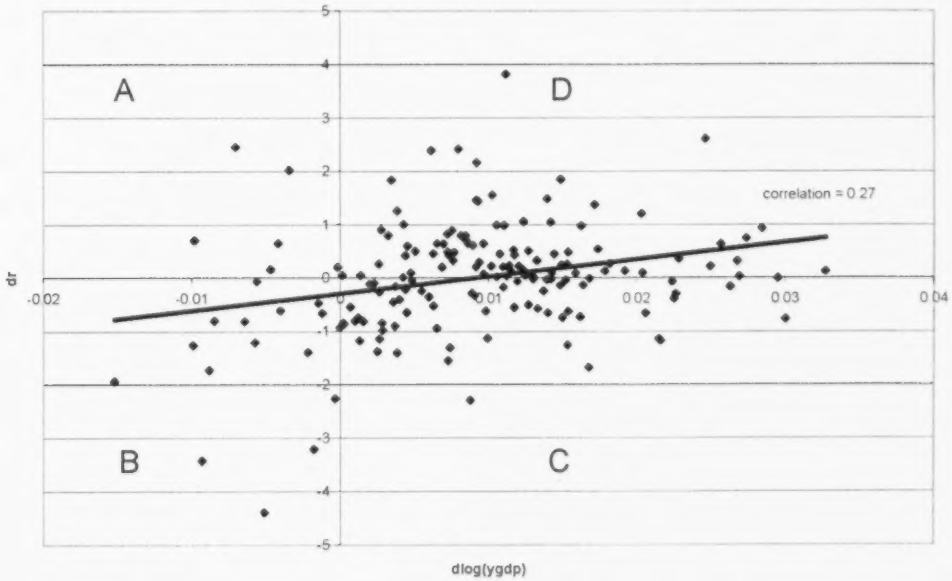
Figure 6b: Risk of avoiding a downward pressure on the debt service cost when the economy is weak



The bottom line for the policy maker is to ask whether the government is more likely to be faced in the future with a positive or negative covariance between the economic activity and the interest rates, or, in other words, with demand or with price/interest rate/supply shocks? Figure 6c sheds some light on this issue by illustrating the correlation pattern between quarterly changes in short-term interest rates (the 90-day Canadian commercial paper rate) and growth rates of the economy (based on the Chained

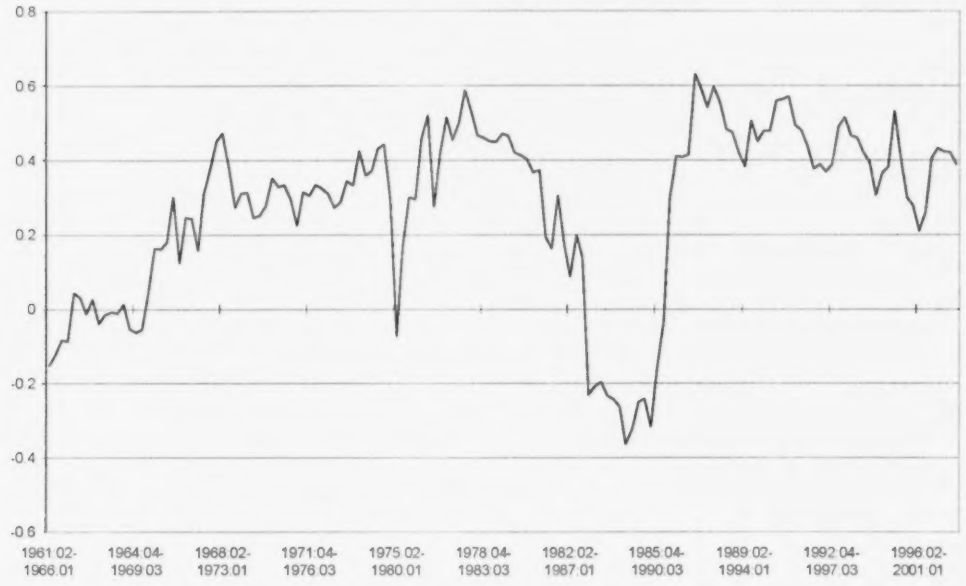
1997 dollar gross domestic product) for the period 1961Q2 to 2002Q3. Coordinates in quadrants B or D are likely to correspond to periods of demand shocks whereas coordinates in quadrants A or C are likely to correspond to periods of non-aggregate demand shocks price/interest rate/supply shocks. The correlation coefficient is positive, but quite low ($r = 0.27$). We observe that when the economy is weakening (negative growth rates) the interest rate (and thus the debt service) is more likely to decrease (quadrant B) than to increase (quadrant A). This seems to suggest that negative demand shocks have been more predominant than other negative shocks.

Figure 6c: correlation between growth rates and changes in interest rates



To convey some dynamics in the correlation pattern over time, Figure 6d illustrates a five-year rolling correlation between the growth rate of the economy and the change in interest rates. The five-year rolling correlation has been clearly positive over most of the period with the main negative correlation observed in the early- to mid- eighties. This again suggests that demand shocks have been predominant over the medium-term (5-year horizon).

Figure 6d: 5-year rolling correlation



Returning to our *simulated* results, Figures 4a and 4b show that our model generates an "average" positive covariance between the primary balance and debt service cost. This positive covariance reflects the calibration of our stochastic simulation model – fitting the model to the data by matching moments (*e.g.*, variances) of variables simulated by the model to empirical moments observed in Canadian data observed over recent history when inflation was low and stable. Hence, aggregate demand shocks are more prominent than inflation and interest shocks in the *simulated* horizon, which is supported at first glance by the correlation analysis above.

Why decreasing the fixed-floating ratio now?

Studies supporting any particular debt maturity structure should pay attention, as we do in our analysis, to the sources of shocks that are likely to affect the Canadian economy. Our analysis supports a new lower fixed-floating ratio and thus a shortening of the current maturity structure of the debt if demand shocks are expected to prevail in the future.

Why was it appropriate to increase the fixed-rate portion of the debt in the mid 1990s and now to lower it? First, concerns for exogenous interest rate shocks that led to an increase in the fixed-rate portion of the debt in mid-1990s are now less prevalent than they were in the early 1990s. The improved economic and fiscal policies make the Canadian economy less subject to interest rate shocks originating from external crisis and/or domestic market

disruptions. Second, the reduction of the debt itself has reduced the impact that interest rate shocks may have on the debt and thus on the debt service cost.[10]

A lesser concern for exogenous interest rate and price shocks indicates that demand shocks may possibly dominate in the future. There is, however, no guarantee that this will indeed be the case. This shows the importance of remaining prudent when deciding on shortening the current maturity structure of the debt.

3. A more sophisticated fiscal planning strategy

The fiscal authority in the model plans a budget at the beginning of each fiscal year. After the budget is set, stochastic shocks impact on program spending, tax revenues and debt service, causing the overall budget and thus the debt to diverge from projected levels. In the subsequent fiscal year, the fiscal authority reformulates its budget plans taking into account economic and fiscal developments. A budget surplus of $4 billion is planned every year, which reflects a prudence factor.

Hence, our model mimics both the economic and fiscal outlooks that a fiscal authority must undertake before tabling a budget. Past shocks have different persistence implications on the path of economic variables (inflation, growth rate of real GDP, interest rates) from which fiscal paths (tax receipts, cyclical program spending, and debt service) are projected, and eventually resulting in a revision of the discretionary program spending component in order to maintain a budget plan of $4 billion and thus a projected debt reduction of $4 billion.

According to this strategy, program spending (with the exception of the cyclical component) is entirely discretionary. In this section we envisage a more realistic fiscal planning framework introduced by Georges and Moreau (2002). In particular, we consider that the fiscal authority can commit to some minimum level of program spending over the medium-term fiscal planning horizon. Hence, this floor on program spending can be viewed as a permanent level of program spending. We also allow for an upward revision of this floor when economic conditions permit. Thus, during each fiscal year, the fiscal authority decides whether the nature of past "positive" shocks permit a permanent or a transitory increase in program spending. Box 1 describes this fiscal planning strategy in further detail. We show that the introduction of a permanent component of program spending, below which the government commits spending not to fall, results in the fiscal authority having a limited scope to reduce program spending and may have to plan a budget with a lower level of fiscal prudence than initially desired after negative shocks. This increases the probability of a deficit outcome by skewing (to the left) the distribution of the budget balance.

---

[10] This fact is easily shown in an equation for the dynamics of the debt-to-GDP ratio: $\Delta(debt/ygdp) = (r-g)debt(-1)/ygdp - pbal/ygdp$. This equation shows that the dynamics of the debt are driven by the difference between interest and growth rates (r-g), and more specifically by the product of that differential and the level of accumulated debt. Hence, the sensitivity of the annual change in the debt-to-GDP ratio to variations of the interest-growth rates differential increases with the level of debt.

We argue in this section that this more sophisticated fiscal planning strategy does not affect our initial recommendation, that is, it is still preferable for the fiscal authority to adopt a shorter debt maturity structure. Figures 7 to 10 are equivalent to Figures 1 to 3 and 5 but the simulation results are now based on the more sophisticated fiscal planning strategy.

Figures 7 to 9 show that the nature of the trade-off perceived by debt managers remains unchanged: a shorter debt maturity structure will reduce average debt service costs but at the cost of a more risky debt service distribution. Figure 10 shows that lowering the fixed-floating ratio remains a strategy that lowers the probability of a deficit outcome. What drives this result is again the average positive covariance between primary balance and debt service, which reduces the variance of the overall budget balance. However, this is not the full story. To this "variance effect" must be added a reduced average skewness of the overall budget balance. Indeed, as explained in Box 1, if (negative) demand shocks predominate, the lower debt service that comes with a lower fixed-floating ratio requires smaller adjustments (cuts) in discretionary program spending to maintain a given level of fiscal prudence. Hence, the "floor" on program spending is less often binding, which reduces the skewness (the long tail to the left) that such a binding floor implies for the distribution of the overall budget balance and thus also reduces the probability of a deficit outcome.

A comparison of Figures 5 and 10 suggests that lowering the fixed-floating ratio under the more "sophisticated" fiscal planning scenario reduces the probability of a deficit outcome by a magnitude roughly similar to that achieved under the fiscal planning scenario considered in Section 2. For example, the vertical distance between the paths for the probability of a deficit outcome for the 64% and 50% fixed-floating ratios is only marginally larger in Figure 10. Hence, relative to the reduced "variance effect", the reduced "skewness effect" due to a lowering of the fixed-floating ratio contributes only marginally to the reduced probability of a deficit outcome.

Box 1. A more sophisticated fiscal planning scenario: adding skewness to the variance effect.

Relative to the fiscal planning scenario of Section 2, the more "sophisticated" fiscal planning scenario of Section 3 adds two elements. The first element is a floor on program spending that serves as a pre-commitment by the fiscal authority to a minimum level of program spending with no option to backtrack. The second element is a conditional rule that enables the fiscal authority to enrich this floor.

In the simple fiscal planning rule of Section 2 the fiscal planner considers all components of the budget as discretionary (apart from a small cyclical component) and thus can always decrease program spending (or increase the tax rate) and table for the next fiscal year a budget surplus of, say, $4 billion (reflecting an amount of fiscal prudence). For example, the table below shows how the fiscal authority would reduce discretionary program spending (PSP) in the next budget as a fiscal response to negative demand or interest rate shocks, and depending on the fixed-floating ratio.

	Negative demand shock	Negative interest rate shock
High fixed-floating	Tax ↓, DS↓, PSP↓↓	Tax ↓, DS↑, PSP↓↓↓
Low fixed-floating	Tax ↓, DS↓↓, PSP↓	Tax ↓, DS↑↑, PSP↓↓↓↓

In the case of a negative demand shock, and with a high fixed-floating ratio, tax revenue decreases, debt service decreases following a lowering of the interest rate by the central bank, but this remains a modest fall because a large part of the debt is fixed, and hence, residually, the government should decrease discretionary program spending substantially to maintain a prudence of $4 billion in the next budget. With a low fixed-floating, tax revenue decreases, debt service decreases more substantially because a large share of the debt is floating, and hence the government need not reduce discretionary program spending as much as before in order to maintain a fiscal prudence of $4 billion in the next budget.

In the case of a negative interest rate/price shock and with a high fixed-floating ratio, tax revenue decreases, debt service increases slightly because the higher interest rate affects only a small floating debt, and so discretionary program sending does not need to decrease as much as it would under a lower fixed-floating ratio in order to maintain a prudence of $4 billion.

The table above does not take into account any constraint on the discretionary program spending component of the budget. In reality, however, there is a limit to the discretionary room available to the fiscal planner; for example, there is a limit to cuts in program spending. The floor on program spending in our more "sophisticated" fiscal scenario simply reflects the existence of this limit to the discretionary room available to the fiscal planner. In this case, following a sequence of negative shocks that are expected to persist for a while, the fiscal authority might have to table a budget with lower tax revenues without being able to reduce significantly program spending, which lowers the amount of fiscal prudence in the next budget (*i.e.*, prudence is less than $4 billion), and which increases the probability of a deficit outcome. This increased probability of a deficit outcome is not due to a change in the second moment (variance) of the probability distribution of the overall balance but due to the skewness (long tail to the left) of this distribution when a floor on program spending is introduced. The fact that the probability of a deficit outcome tends to increase more through time (for all fixed-floating ratio strategies) in Figure 10 than is the case in Figure 5 is due to the pre-commitment on program spending or limited discretionary room available to the fiscal planner.

Based on the table above, we can see that if demand shocks predominate over the business cycle, lowering the fixed-floating ratio will increase the covariance between tax and debt service, which decreases the variance of overall balance, and will reduce its skewness (to the left). Both effects act to lower the probability of a deficit outcome when the fixed-floating ratio is decreased. The reduced skewness does not contribute to a significant lower probability of a deficit outcome. Indeed, the vertical distance between the paths for the probability of a deficit outcome for the 64% and 50% fixed-floating ratios increases only slightly in Figure 10 compared to Figure 5.

(…)

(…)

The second element of our more "sophisticated" fiscal planning scenario is a fiscal rule that enables the fiscal authority to enrich the minimum floor on program spending. In our model, enrichments to the floor are conditional on the arrival of positive economic shocks. The magnitude of the enrichment is such that it maintains the probability of a deficit outcome unchanged relative to what it was before the arrival of the economic shock. To illustrate, suppose that a positive permanent shock was not followed by any permanent enrichment in program spending (or tax cuts). This would increase the expected fiscal surplus (or discretionary fiscal room) and thus would lower the probability of a deficit outcome for the future years. On the other hand, if the fiscal authority were over-ambitious in its enrichment, this would decrease the expected surplus and thus increase the probability of a deficit outcome. Our conditional rule is calibrated to enrich the floor on program spending by the amount that would maintain the scope for reducing program spending, and thus the probability of a deficit outcome to what it would have been prior to the positive shock.

In this framework, the fixed-floating ratio per se does not affect the performance of the conditional fiscal rule in terms of probability of a deficit outcome. Independently of the fixed-floating ratio, the rule enriches permanent program spending or introduces permanent tax cuts by the amount that leaves the probability of a deficit to what it would have been prior to the arrival of the positive shock. (In other words, the amount of fiscal measure introduced might be affected by the choice of the fixed-floating ratio, but not the probability of a deficit outcome.) Hence, the conditional fiscal rule of our more sophisticated fiscal framework does not generate any additional effect that offsets or amplifies the variance and skewness effects described above.

Figure 7. Average Debt Service Cost Under Alternative Debt Maturity Structures

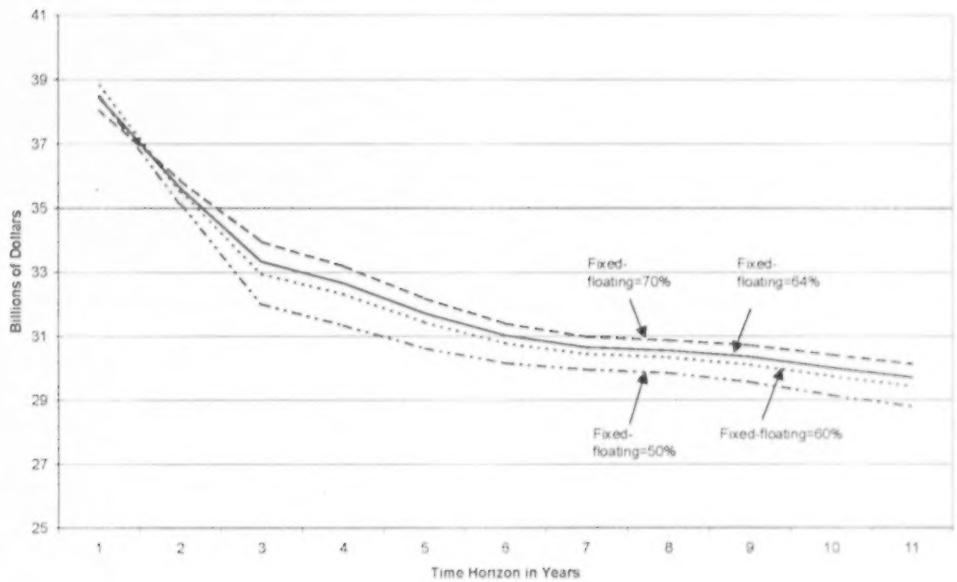


Figure 8. 90% Confidence Intervals for Debt Service Costs

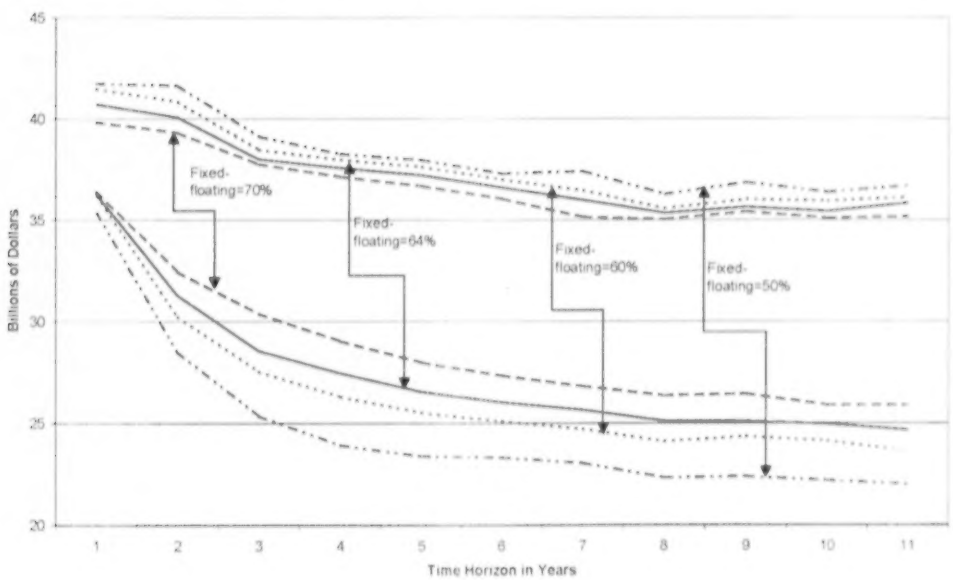


Figure 9. Relative Cost at Risk

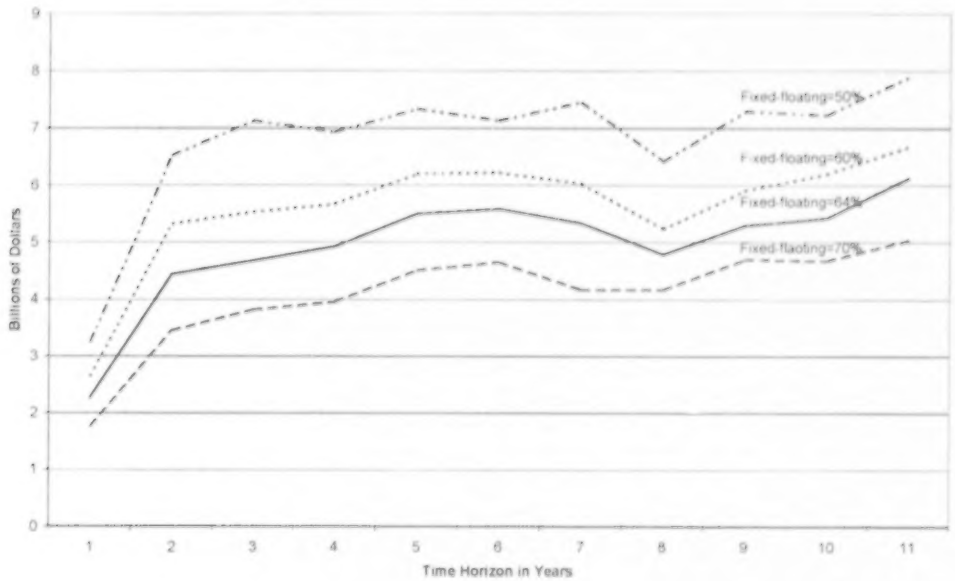
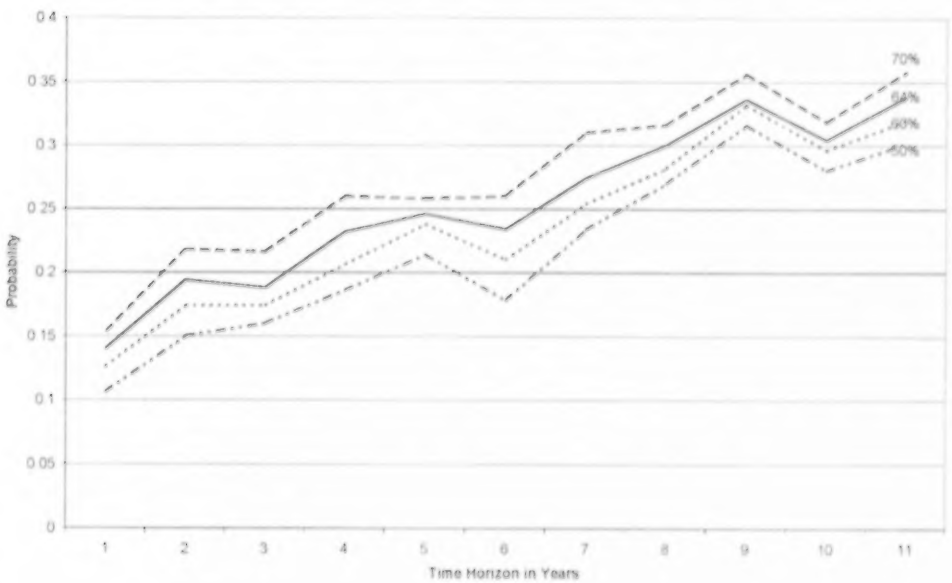


Figure 10. Probability of a Deficit Outcome Under Alternative Debt Maturity Structures



4. Issues, caveats and next steps

a. Is the central bank reaction function key to our results?

A legitimate question is whether our results would hold in absence of a central bank reaction function. We argue that even in this case it is still preferable to select a short debt maturity structure.

For example, in the case of an expansionary aggregate demand shock, if the monetary authority does not respond by raising interest rates to curtail inflationary pressures, then inflation would be maintained at a higher level. This results in higher *nominal* interest rates. On one hand, if the debt maturity structure were biased in favour of Treasury Bills and short-term bonds, this would increase the nominal debt service payment because this period's refinancing of a large share of the debt is done at higher nominal interest rates. In terms of equation (2) above, this also suggests a large covariance between the primary balance and the debt service and a stabilizing effect of the debt service on the overall budget balance.

On the other hand, if the debt maturity structure were biased towards longer-term bonds, the increase in nominal interest rates due to inflationary pressures would have relatively little impact on this period's debt service payment because only a small share of the public debt would have to be refinanced this period at higher nominal rates. The stabilizing effect of the debt service payment on the overall balance would thus be small in this scenario.

In conclusion, even absent a central bank reaction function, the fiscal authority is likely to prefer a shorter maturity structure of the debt because this structure permits them to exploit the stabilizing impact of the debt service on the overall budget balance and thus on the probability of a deficit outcome.

b.Does the permanent income hypothesis underlying a "supply" shock affect our results?

As mentionned in Section 2, shocks to trend or potential output ("supply" shocks) are modeled on the assumption of a permanent income hypothesis. For example, a decrease in potential output leads to an immediate and equal decrease in economic activity without generating an output gap and thus, without generating pressures on prices. Hence, there is no reason for the central bank to react by revising its policy interest rate. The decrease in potential output decreases the primary balance through lower tax revenues, but with no change in interest rates. Hence, in our model, this kind of "supply" shock implies a zero covariance.

However, as described in the OECD Report (2002) a supply shock can also affect the output gap if demand does not immediately adjust entirely to supply. In this context, a negative shock to potential output does not translate immediately into a one-to-one decrease in economic activity, thus generating a temporary positive output gap (demand above potential), although the level of the economic activity is below its level prior to the

shock. Hence, tax revenues and the primary balance will decrease at a time when the central bank increases its policy interest rate to close the positive output gap. This generates a negative covariance between debt service and the primary balance. Introducing this feature in our model would add to the negative covariance of inflation and price shocks and thus would mitigate our policy conclusion for a shorter debt maturity structure. We are now in the process of determining whether we can easily introduce this feature in our model.

c. Do we model the term structure of interest rates adequately?

Debt managers may ask whether our results depend on our assumptions concerning the modelling of the term structure of interest rates. Best practises of debt management require the use of modern theories of the term structure based on the seminal papers by Vasicek (1977) and Cox, Ingersoll and Ross (1985). These models have been used to analyse the issue of public debt management both at the Bank of Canada [Bolder (2002)], and at the Department of Finance [Debt Management Strategy 2003-2004], and in other countries [*e.g.*, Danish Nationalbank (2001)].

In Georges (2003), we show that these models belong to the class of the biased expectations hypothesis of the term structure of interest rates. This paper demystifies these models to the non-experts of the field. Indeed, what they imply is that the long interest rate is an average of future expected short rates plus a term premium. In models à la Vasicek, the expected future short rates are consistent with an exogenously specified process for the short rate. In our model, the term structure also belongs to the class of the biased expectations hypothesis, but the expected future short rates are fully consistent with the macro model of the Canadian economy.

However, to test the robustness of our results we also specified alternative term structures for the interest rates in our stochastic model, one that corresponds to the one-factor Vasicek specification and others that correspond to the one and two factors Cox-Ingersoll-Ross specifications. One key issue, however, is the interaction of the central bank reaction function and the exogenously given process for the short interest rate. There are alternative ways to think of this issue and this should be considered in future research.

d. Could the debt financing strategy be self-defeating?

The traditional literature takes the average slope of the yield curve as given (from historical behaviour of interest rates) and does not ask how does debt management policy might affect interest rates. Some authors (*e.g.*, Agell, Persson and Friedman, 1992) have argued, however, that if the government decided to borrow heavily in Treasury Bills and short-term bonds while turning away from long-term borrowing, the increased supply of Treasury bills would drive down the price of bills relative to the prices of long-term bonds. This would imply an increase in short rates relative to the long interest rates. Hence, the slope of a typical yield curve might well flatten as the debt maturity structure is shortened. This raises the issue of whether a government that shortens the debt

maturity structure can effectively decrease the average debt cost. We can envisage a strategy so biased in favour of short-term instruments that the average debt service cost could actually *rise* due to higher pressures on short rates eliminating the sought-after savings. Campbell (1995) calls this a self-defeating strategy. In this case the recommendation given above that a government should prefer the shortest among the feasible debt maturity structure might well become self-defeating. Although the probability of a deficit outcome would decrease, the average debt service cost might well increase. Our preliminary discussions with debt managers seem to suggest that this effect, if at all, is small, and that it is very unlikely that this strategy could become self-defeating for small changes in the fixed-floating ratios.

A modelling of the yield-curve slope reflecting this feedback from the debt maturity structure could provide a non-corner solution when choosing a debt maturity structure. The impact of the financing strategy on the slope of the yield curve in unclear, however, and more research is needed before progressing along these lines. Although it is easy to model on an ad hoc basis the slope of the yield curve as a function of the debt maturity structure (for example, by recalibrating the term premiums), the non-corner solution that we would obtain would simply reflect our arbitrary assumptions.

e. Could fiscal prudence be increased when the fiscal authority reduces the fixed-floating ratio?

In our model, the lower average debt service cost associated with a lower fixed-floating ratio is immediately reflected in a higher average program spending profile or a lower average tax revenue profile. This ensures that the fiscal prudence (or contingency reserve) remains identical for all fixed-floating ratios and that the debt is reduced year after year on average by the same amount (the amount of prudence). Alternatively, all savings associated with a lower average debt service cost could be allocated to fiscal prudence such that on average, public debt would be reduced faster under a shorter maturity structure. The increased prudence due to the lowering of the fixed-floating ratio would reduce the probability of a deficit outcome as a first order effect, and the faster debt reduction might also contribute, as a second-order effect, to reducing this probability.

5. Conclusion

This study does not claim to provide the last word on the issue of debt management as should be obvious from the non-exhaustive list of caveats and further issues given in Section 4. Instead, this paper illustrates, for the Canadian case, a new approach to the issue of debt management that was recently suggested by an OECD Report (2002).

Debt management has often highlighted the existence of a trade-off between the first and second moments of the debt service distribution (average cost and variance) generated within a purely financial model. A fiscal planner perspective, however, by embracing a more global perspective that includes the interdependence of macro and financial variables may considerably change our way of thinking about the issue of debt

management. Indeed, a fiscal planner would be more concerned with the risk that debt service cost rises at a time when the budget is already weak due to other factors. This shows the importance of increasing our understanding of the covariance between debt service costs and the primary balance and the relevance of developing models (such as ours) which include a set of macro-variables that interact with financial variables. Purely financial models are unable to shed light on this specific issue because they do not consider these macro-developments.

Our preliminary results suggest that the bottom line for the policy maker is to ask whether the government is more likely to be faced in the future with a positive or a negative covariance between the primary balance and the debt service? In our model, the federal government is more likely to be faced with a positive covariance. The average positive covariance that we generate by simulating our model reflects the calibration of our stochastic simulation model to match the moments observed over recent history when inflation was low and stable. As a consequence, aggregate demand shocks are more prominent than inflation shocks for the simulated horizon, which generates this average positive covariance and underlies our conclusion in favour of a shorter maturity structure for federal debt. However, there is no guarantee that the covariance will indeed be positive in the future. This shows the importance of remaining prudent when deciding on shortening the current maturity structure of the debt.

Appendix 1

1. The maturity structure of the debt

We approximate the maturity structure of federal debt by focusing on eight maturity categories ranging from one quarter (90-day treasury bills) to 20 years. We abstract from considerations associated with having foreign-currency denominated debt, retail debt instruments and investment income earned on financial assets held by the federal government.

The weights associated with maturities in the federal debt structure are given in the second column. The decomposition between floating and fixed shares is given in the last two columns. The last row of Table 1 indicates that 36% of the federal debt must be refinanced every year, and thus that 64% of the debt is fixed (*i.e.*, does not mature within a one-year period).

Table 1. The maturity structure of the debt

Maturity, m	Weight in debt	Floating (maturing in 1 year or less)	Fixed (maturing beyond one year)
3 months	$G_1 = 0.11$	0.11	0
6 months	$G_2 = 0.08$	0.08	0
1 year	$G_3 = 0.06$	0.06	0
2 years	$G_4 = 0.03$	(1/2)*0.03	(1/2)*0.03
3 years	$G_5 = 0.06$	(1/3)*0.06	(2/3)*0.06
5 years	$G_6 = 0.22$	(1/5)*0.22	(4/5)*0.22
10 years	$G_7 = 0.22$	(1/10)*0.22	(9/10)*0.22
20 years	$G_8 = 0.22$	(1/20)*0.22	(19/20)*0.22
	$\sum G_i = 1$	$\sum w_{\text{floating}} G_i = 0.36$	$\sum w_{\text{fixed}} G_i = 0.64$

Lowering the fixed rate share of the debt to, say, 50%, implies issuing less long-term maturity instruments and more short-term maturities. There are obviously a large number of financing strategies that may correspond to the same fixed-floating ratio.

There exist alternative measures of the maturity of the debt, for example the average term to maturity, duration, etc. [See for example, Copeland and Weston (1992).] We do not look at these other measures in the paper.

2. Debt service costs

Debt service payments are analysed with reference to the implicit interest rate on net federal debt. The implicit rate can be computed over the historical period as:

$$i_t^{nd} = \frac{DS_t}{ND_t}$$

where i_t^{nd} is the implicit interest rate on net federal debt, DS_t is the value of debt service payments made in period t, and ND_t is the value of outstanding net federal debt.

The implicit interest rate on net federal debt is endogenous in the simulation model. It is calculated as a weighted average of current and lagged interest rates on government securities with different dates. The weights reflect two factors. First, new debt issues are financed by bonds with different maturities. Second, interest rate payments are made on outstanding bonds that were issued at different dates in the past. The implicit interest rate on net federal debt, i_t^{nd} is calculated as follows:

$$i_t^{nd} = G_1 i_{t,t+1} + G_2 \frac{i_{t,t+2} + i_{t-1,t+1}}{2} + G_3 \frac{i_{t,t+4} + i_{t-1,t+3} + i_{t-2,t+2} + i_{t-3,t+1}}{4}$$
$$+ G_4 \frac{i_{t,t+8} + i_{t-1,t+7} + i_{t-2,t+6} + i_{t-3,t+5} + i_{t-4,t+4} + i_{t-5,t+3} + i_{t-6,t+2} + i_{t-7,t+1}}{8}$$
$$+ G_5 \frac{\sum_{j=0}^{11} i_{t-j,t+12-j}}{12} + G_6 \frac{\sum_{j=0}^{19} i_{t-j,t+20-j}}{20} + G_7 \frac{\sum_{j=0}^{39} i_{t-j,t+40-j}}{40} + G_8 \frac{\sum_{j=0}^{79} i_{t-j,t+80-j}}{80}$$

where $i_{t-j,t+m-j}$ represents the yield on a m-maturity bond issued in period t-j and maturing in period $t+m$-j where t is the current period (such that if j=0 we are considering newly issued bonds whereas for positive j we are considering bonds issued j periods ago). The weights G_i represent the proportion of outstanding bonds of each maturity m as given in Table 1.

The logic of the above equation is as follows. The implicit interest rate should avoid taking into account yields of previously matured instruments, but should keep track of all outstanding bonds. For example, at time t all previously issued bonds of 3-month maturity have matured such that yields $i_{t-1,t}$, $i_{t-2,t-1}$… should be excluded from the implicit interest rate computation. We are thus left with the newly (time-t) issued 3-month maturity bond with yield: $i_{t,t+1}$. This also holds true for 6 months bonds issued 6 months ago or earlier such that yields $i_{t-2,t}$, $i_{t-3,t-1}$… should be excluded. We are thus left with two outstanding classes of bonds with a 6-month maturity, those that have just been issued this period, and those that were issued 3 months ago. The implicit interest rate for a 6-month bond is then assumed to be the average of these two yields: $\frac{i_{t,t+2} + i_{t-1,t+1}}{2}$. The same reasoning applies for all other maturities.

References

Agell, Jonas, Mats Persson, and Benjamin Friedman (1992), *Does Debt Management Policy Matter*? Oxford: Oxford University Press.

Bolder, David (2002): "The Government of Canada's Debt Strategy Simulation Framework", Discussion paper, Bank of Canada.

Campbell, John (1995): "Some Lessons from the Yield Curve", *Journal of Economic Perspectives*, Vol. 9, No. 3, pp.129-152.

Clark, P.K. (1987): "The Cyclical Component of U.S. Economic Activity", *Quarterly Journal of Economics*, Vol.102, No.4, pp. 797-814.

Copeland and Weston (1992): *Financial Theory and Corporate Policy*. Reading, Massachusetts: Addison-Wesley Publishing Company.

Cox, John, Jonathan Ingersoll, and Stephen Ross (1985): "A Theory of the Term Structure of Interest Rates", *Econometrica*, Vol. 53, No. 2, pp. 385-407.

Danish Nationalbank (2001): "Danish Government Borrowing and Debt 2001", available at: http://www.nationalbanken.dk/nb/nb.nsf/alldocs/Sdanish_government_debt_fullte xt_2001/$File/Chap09.htm

Department of Finance-Canada: "Debt Management Strategy", various years, available at http://www.fin.gc.ca/purl/dms-e.html

Georges, Patrick and Nicolas Moreau (2002): Enrichments to Fiscal Commitments in the Presence of Uncertainty, Working Paper # 2002-11, Department of Finance-Canada.

Georges, Patrick (2003): "The Vasicek and CIR Models and the Expectation Hypothesis of the Term Structure", Working Paper # 2003-17, Department of Finance-Canada.

Hermanutz, Derek and François Poitras (2001): "The Fiscal Impact of a Debt Maturity Structure", Discussion paper, Department of Finance-Canada.

Hostland, Doug (2001): "Specification of a Stochastic Simulation Model for the Analysis of Monetary and Fiscal Policy", Department of Finance-Canada Working Paper # 2001-14.

OECD (2002): "Report on Interest-Rate Risk Management of Government Debt", Prepared by Ad hoc Working Group on Interest-Rate Risk Management of Government Debt.

Vasicek, Oldrich (1977): "An Equilibrium Characterization of the Term Structure", *Journal of Financial Economics* 5, pp. 177-188.